

NONEQUILIBRIUM CHIRAL PERTURBATION THEORY¹

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Abstract

We explore the extension of chiral perturbation theory to a meson gas out of thermal equilibrium. For that purpose, we let the pion decay constant be a time-dependent function and work within the Schwinger-Keldysh contour technique. A useful connection with curved space-time QFT is established, which allows to consistently renormalise the model. We discuss the applicability of our approach within a heavy-ion collision environment

The nonequilibrium dynamics of the chiral phase transition has attracted considerable interest during the last few years. One of the original motivations to analyse this regime was the suggestion [1] that the so called disoriented chiral condensates (DCC) could form during the plasma expansion after a relativistic heavy-ion collision (RHIC), giving rise to observable effects such as coherent pion emission [2]. Traditionally, this and other similar effects have been investigated using $O(N)$ models in which initial thermal equilibrium is assumed and nonequilibrium is parametrised as the time dependence of the different lagrangian parameters [2, 3, 4, 5]. Such time dependence is indeed the only relevant nonequilibrium feature when working for instance within Bjorken's initial conditions approach, where observables such as the order parameter depend only on proper time in the central rapidity region [6].

The use of effective models for QCD is imperative to describe properly the dynamics of the plasma in this regime of temperature and energy density. The $O(N)$ models include explicitly the σ meson and are valid only for two light flavours ($N_f = 2$). Besides, for strongly coupled systems, perturbation theory in these models is not well defined and one needs to use resummation methods such as large N . An alternative, not so well investigated, is to use the effective chiral lagrangian formalism, whose main advantage is to provide a consistent perturbation theory in powers of p/Λ_χ —Chiral Perturbation Theory (ChPT) [7]—where p denotes generically any meson external momenta or field derivative, and $\Lambda_\chi \simeq 1$ GeV. Besides, it is also valid for $N_f = 3$. So far, this formalism has been applied only in thermal equilibrium, to study the low T ($T = \mathcal{O}(p)$) meson gas and the chiral phase transition [8, 9].

We will review here recent work [10] on the extension of ChPT to a nonequilibrium situation. The key idea is to make use of the derivative expansion naturally incorporated in ChPT, in order to study the system not far from equilibrium. Of course, in a later stage one could apply resummation methods such as large N so as to extend the validity of the approach closer to the critical point, as it has already been done in equilibrium [9].

Our starting point is the nonlinear sigma model (NLSM) where we let the pion decay constant—the only relevant parameter to the lowest order in derivatives—be time dependent.

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As commented above, such time dependence can be thought of as proper time evolution. We take the initial time $t = 0$, which would correspond to a proper time $\tau_0 \simeq 1 \text{ fm}/c$, a typical hadronisation time in a RHIC environment [6]. Thus, we will consider the NLSM action

$$S[U] = \int_C dt \int d^3\vec{x} \frac{f^2(t)}{4} \text{tr} \partial_\mu U^\dagger(\vec{x}, t) \partial^\mu U(\vec{x}, t) \quad (1)$$

Here, C is the Schwinger-Keldysh contour in time, containing an imaginary leg of length $\beta_i = 1/T_i$, $T_i < T_c$ being the temperature for $t \leq 0$, where we assume that the system is in thermal equilibrium. Thus, $f(t \leq 0) = f_\pi \simeq 93 \text{ MeV}$ to leading order. For $t > 0$ the system departs from equilibrium. Note that, since we choose that departure to be instantaneous, $f(t)$ cannot be analytical at $t = 0$. This feature can give rise to discontinuities in the observables, and even extra singularities at $t = 0$, as it has been noted in [11]. Finally, in the above equation, $U(x)$ is the Goldstone boson field, which we can parametrise as customarily in terms of pions for $N_f = 2$ plus kaons and eta for $N_f = 3$ [10]. We shall restrict here to $N_f = 2$.

The new ingredient we need to incorporate in the power counting in order to be consistent with ChPT is

$$\frac{\dot{f}(t)}{f^2(t)} \simeq \mathcal{O}\left(\frac{p}{\Lambda_\chi}\right), \quad \frac{\ddot{f}(t)}{f^3(t)}, \frac{[\dot{f}(t)]^2}{f^4(t)} \simeq \mathcal{O}\left(\frac{p^2}{\Lambda_\chi^2}\right), \quad (2)$$

and so on. Otherwise, we shall keep $f(t)$ arbitrary. One can think of $f(t)$ as an external source, to which we will find the nonequilibrium response of the system. Another alternative, which we will not attempt here, is to treat $f(t)$ as a field and solve for $f(t)$ the hydrodynamic equations self-consistently.

Once we have defined our nonequilibrium power counting, we can apply ChPT to calculate the time evolution of the different observables. In doing so, we must pay special attention to renormalisation. In standard ChPT, one-loop UV divergences coming from the $\mathcal{O}(p^2)$ lagrangian are canceled by tree level contributions coming from the $\mathcal{O}(p^4)$ one, and so on for higher order contributions. Such fourth-order action is the most general one preserving all the symmetries. On the other hand, it is a well-known feature of nonequilibrium field theories that new infinities (time dependent in this case) can arise [11, 12]. Thus, our fourth-order lagrangian must contain necessarily new terms, to account for extra divergences. In order to find them, we will make use of a very fruitful analogy: the action (1) is equivalent to formulate the NLSM on a curved space-time background corresponding to a spatially flat Robertson-Walker metric, with scale factor $a(t) = f(t)/f(0^+)$ [10]. In this way, we can construct the $\mathcal{O}(p^4)$ lagrangian in the following way: we just raise and lower indices with our RW metric in the standard (equilibrium) terms and add those terms coupling $U(x)$ with the scalar curvature $R(x)$ and the Ricci tensor $R_{\mu\nu}(x)$ to this order. The latter involve new low-energy constants that are fixed by analysing the energy-momentum tensor of QCD at low energies [13].

As a first application of our approach, we have analysed the pion decay functions (PDF) to next to leading order, i.e, including loops with (1) and tree level diagrams from the $\mathcal{O}(p^4)$ lagrangian. In thermal equilibrium, one can define two pion decay constants, $f_\pi^s(T)$ (spatial) and $f_\pi^t(T)$ (temporal) due to the loss of Lorenz covariance [14]. The same

happens out of equilibrium, where those constants turn into time-dependent functions. After analysing the corresponding Feynman graphs the result can be written as [10]

$$[f_\pi^s(t)]^2 = f^2(t) [1 + 2f_2(t) - f_1(t)] - 2iG_0(t) \quad (3)$$

$$[f_\pi^t(t)]^2 = f^2(t) [1 + f_2(t)] - 2iG_0(t) \quad (4)$$

for $t > 0$, where

$$\begin{aligned} f_1(t) &= 12 \left[(2L_{11} + L_{12}) \frac{\ddot{f}(t)}{f^3(t)} - L_{12} \frac{[\dot{f}(t)]^2}{f^4(t)} \right] \\ f_2(t) &= 4 \left[(6L_{11} + L_{12}) \frac{\ddot{f}(t)}{f^3(t)} + L_{12} \frac{[\dot{f}(t)]^2}{f^4(t)} \right] \end{aligned} \quad (5)$$

Here, L_{11} and L_{12} are the two new low-energy constants that need to be introduced to this order [13]. While L_{12} is already finite, L_{11} needs to be renormalised in dimensional regularisation. In (3)-(4), $G_0(t)$ is nothing but the equal-time pion two-point function $G_0(t) = G_0(x, x)$ with $G_0(x, y)$ the solution of the differential equation

$$\{\square_x + m^2(x^0)\} G_0(x, y) = -\delta_C(x^0 - y^0) \delta^{(3)}(\vec{x} - \vec{y}) \quad (6)$$

with KMS equilibrium conditions $G_0^>(\vec{x}, t_i - i\beta_i; y) = G_0^<(\vec{x}, t_i; y)$, $t_i < 0$ and $m^2(t) = -\ddot{f}(t)/f(t)$ plays the role of a time-dependent mass. It is important to remark that this mass is a consequence solely of the nonequilibrium behaviour and has nothing to do with an explicit chiral symmetry breaking pion mass term. In fact, our model is exactly chiral invariant, since, for simplicity, we have chosen to work in the chiral limit (there are no pion mass terms in (1)). In the language of curved space-time QFT, $m^2(t)$ is the minimal coupling with the metric preserving chiral invariance. Note that $m^2(t)$ can be negative, thus allowing the existence of unstable long-wavelength modes, which play an essential role during the plasma evolution [2, 3, 4, 5].

The results (3)-(4) reproduce the equilibrium $f_\pi(T)$ when we switch off the time derivatives of $f(t)$. We remark that $G_0(t)$ contains UV divergences, giving rise to time-dependent singularities, to be absorbed by $f_1(t)$ and $f_2(t)$ in the renormalisation of L_{11} . An interesting consequence of our result is that $f_\pi^s(t) \neq f_\pi^t(t)$ to one-loop, unlike the equilibrium case [14]. In addition, from (3)-(4) and (5) we see that the difference $[f_\pi^s(t)]^2 - [f_\pi^t(t)]^2$ is finite, so that we can renormalise both at the same time, which is another consistency check.

Therefore, given $f(t)$, all one has to do to this order is to solve the leading order propagator equation (6). Of course, that is not possible in general, as it is well-known in the context of curved space-time QFT, where solutions can be analytically found only for a very few choices of the scale factor, or equivalently, for $f(t)$ [15, 16]. Hence, in order to estimate the nonequilibrium effects within our approach, we shall perform a short-time approximation. We will restrict then to $0^+ < t < t_{max}$ where 0^+ means a small response time and $t_{max} \simeq 1/f_\pi \simeq 2 \text{ fm/c}$. For that range, the expansion in t is also a

chiral expansion, so that we can truncate it to the relevant order. Thus, solving for the propagator by expanding $m^2(t)$ in power series, we get

$$\left[f_\pi^{s,t}(t)\right]^2 = \left[f_R^{s,t}\right]^2 \left\{ 1 - \frac{T_i^2}{T_c^2} + 2Ht - \left[m^2 \left(1 - \frac{T_i^2}{T_c^2} \right) - H^2 \right] t^2 + \mathcal{O}(p^3/\Lambda_\chi^2) \right\} \quad (7)$$

where $T_c = \sqrt{6}f_\pi$, $m^2 = -\ddot{f}(0^+)/f(0^+)$ and $H = \dot{f}(0^+)/f(0^+)$. Notice that unstable modes appear for $m^2 < 0$. Here, the two constants $f_R^{s,t}$ are already renormalised in terms of L_{11} and L_{12} and are fixed by the physical value of $f_\pi^{s,t}(t = 0^+)$. We remark that the $t = 0$ discontinuities are a consequence of our choice of initial conditions. Nonetheless, the effect of the L 's turns out to be relatively small since $L_{11}^r, L_{12}^r \simeq 10^{-3}$ [13].

Within our short-time approach, we can estimate the thermalisation time by $f_\pi(t_f) = f$ (neglecting the effect of the L 's) which is also the freezing time, since f_π reaches its zero temperature value. It is clear that by expanding in t we cannot reproduce a thermalisation process where $T_f \neq 0$. The maximum value $t_f \simeq t_{max}$ is reached for $H < 0$ and $m^2 < 0$. In fact, unstable modes always tend to cool down the system. Our estimates are somewhat lower than typical $O(N)$ calculations [4, 5], which is not surprising, since in those works the $T_i \geq T_c$ and then gradients are too high to compare with our approach. For instance, in [5], $T_i \simeq 200$ MeV and $|H| \simeq 400$ MeV. Thus, we expect our model to be valid when some cooling has already taken place and the system enters the validity range of low-energy ChPT. Nevertheless, it is worth remarking that large N methods would allow us to depart further from equilibrium. Other extensions and applications of our model to be analysed in the future include the long-time evolution – by choosing suitable parametrisations for $f(t)$ –, the $N_f = 3$ case and the formation of DCC. The inclusion of quark masses and gauge fields would allow to investigate, respectively, the quark condensate time dependence and photon production in the pion sector.

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